Elements of Numerical Analysis Used for Data Processing in Sport

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Abstract— **There are more approaches concerning the using of the informatics on sport domain, in the important fields of it, the most important being the analysis of training and competition. The biomechanical analysis instruments, data bases for documentation concerning the training and competition and the video techniques plays an essential part on the systems for studying of the sportsmen as part of teams, on individual sports and on sportive subjects. The scientific innovation of this paper consists in the preparing of one methodology to forecast the sportsmen training through applying of one mathematical modeling method that, through the exponential functions, establishes, with an approximation of maximum 2%, the forecasting of the sportsmen's individual parameters enhancing. In most cases, the extrapolation functions, obtained by computing technique, overlap the real values obtained after a short training period.**

Index Terms—**numerical analysis, data prediction, training process, extrapolation functions**

I. INTRODUCTION

Studying the specialty literature it was observed that in actual epoch, the sportive training process is based on the well organized activities, planned and lead by laws, principles and rules subordinated to biological, psychical and social sides, where are especially observed the progress of the motive aptitudes simultaneous with that intellectual and affective. This process must organized and planned with more attention during the period in which appears a series of biological, physical and psycho-motive changes in the sportsman body.

The large majority of specialists on sport domain, after the undertaken studies, proposed varied solutions to improve this process and promoted modern methods and means. In this way, came into being some installations, equipments and computerized technologies that more contributed to the improvement of the training process development. The computing technique also integrated itself into the selection process and into sportsmen training. Through this it is used multiple programs for physical effort planning and for refreshment. The informational system can forecast next results that will be obtained by the sportsmen.

From the bibliographical analysis process it was constituted an ample supply concerning the organization and evolution of the sportive training at the high performance level, especially at the senior sportsmen. We consider that in the junior stage doesn't lent a sufficient attention for implementation of some methodologies based on informational technique and on the mathematical methods. The application of mathematical methods and of the computing techniques can contribute to training forecasting and to establish the selection, planning and psycho-motive training way, so to make the sportive performances obtaining to be efficient.

The scientific and technological evolution at an unimaginable pace, in the last ten-year period, the coming into being of extremely sophisticated devices changed the sports domain, holding out large variety and high quality ways. If till no long ago the sportsmen training had on the base the experience of a technical team more or less capable, today the computer became a necessity that plays a decisive part in some sports directions.

Sport represents one of the most dynamic social activities that have for an object the human being perfection. Rom this reason appears as justified the specialist's care to effect periodically forecast analyses with the purpose of discovering the evolution tendencies. We refer, in the main, to the performance sports whose special role amplified continuously, phenomenon that determine some specialist to affirm that the apogee will be followed by a crisis or an imminent decline.

II. MATHEMATICAL METHODS FOR FUNCTIONS APPROXIMATION

Because in the large majority of cases the tests direct to a real function of real variable, the approximation of this characteristic, in the specified cases, consists in the approximation of a real function, approximation named interpolation too. The approximation of the certain real function is made by simple and easy utilized functions, especially through implementation of the computing of the values of this function. Because the real function set is a linear dimensional infinite space, while the function sets in which we look for the approximation are dimensional finite spaces, in actual fact, the abstract problem that stand on the base of approximation techniques consists in replacing of one element from a dimensional infinite space by representatives of one dimensional finite space. To can specify the "approximation" notion and to can appreciate the error made through the above specified replacing, the using of "norm" mathematical concept is needed.

Considering a real linear space *X*, we define on it the norm symbolical noted by:

$\| \, \| : X \to \Re$, having the properties (that satisfy the axioms):

$$\begin{aligned} &a) \quad \|f\| \ge 0, \forall f \in X \\ &b) \quad \|f\| = 0 \Rightarrow f = 0 \\ &c) \quad \|\alpha f\| = |\alpha| \cdot \|f\|, \forall \alpha \in \Re, f \in X \\ &d) \quad \|f + g\| \le \|f\| + \|g\|, \forall f, g \in X \end{aligned} \tag{1}$$

If on the linear space *X* it was defined the norm $\| \, \|$, *X* is a normalized linear space. On these terms we could specify the "approximation" of one function from normalized linear space *X*, defining what means a best approximation.

Considering the dimensional infinite functions space *X*, normalized by $\| \, \|$ and X_m a linear subspace of *X*, dimensional finite with $\dim(X_m) = m$, we define the notion of approximation of one function $f \in X$ by function $g_m \in X_m$ considering $\|f - g_m\|$ as a measure of the error that is made if instead of the function f is used the function g_m. It is naturally on these terms to say that g_m is the best approximation of f if:

$$\left\| f - \overline{g}_m \right\| = \inf_{g_m \in X_m} \left\| f - g_m \right\| = \alpha_m \qquad (2)$$

The main question which is mooted consists in the fact that from the given definition doesn't result that always is a best approximation, not that this, in case it exists, is unique. A much delicate problem, that requires approximations in its turn, is the effective computing of the best approximation in case it exists.

In many cases, in the function approximation theory is sufficient a hypothesis flimsier than the existence of one norm on the space X namely is sufficient the existence of one semi-norm on X that is a function noted also $\| \ \| : X \rightarrow \mathfrak{R}$, but satisfying only the axioms a, b and d from (1). The existence and oneness of a best approximation is assured by the next theorems which will be enounced without demonstration.

Generally, when we know data on certain moments of time, we can find a continuous function that approximates the evolution of these data. Now there are known several kinds of approximation functions able to approximate data that have a certain evolution.

Approximation by Lagrange polynomial [1], [2], uses the Lagrange polynomial Ln(x) and is in the form:

$$L_n(x) = \sum_{i=0}^{n} f(x_i) \cdot \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_i}{x_i - x_j} \qquad (3)$$

Newton approximation [3], [4], eliminates the disadvantage of the Lagrange Polynomial which consists in fact that, in case of adding of one additional approximation node, using the divided differences expression, the approximation polynomial is written as:

$$\begin{aligned} L_n(x) &= f(x_0) + [x_0, x_1] f(x - x_0) + \\ &+ [x_0, x_1, x_2] f(x - x_0)(x - x_1) + \ldots \\ &+ [x_0, x_1, \ldots, x_{n-1}, x_n] f(x - x_0)(x - x_1) \ldots (x - x_{n-1}) \end{aligned} \qquad (4)$$

named also Newton interpolation formula.

Square average approximation [3], [5]. It is easy verified that will be a best unique approximation as polynomial form:

$$\overline{P}_n(x) = \sum_{j=0}^{n} a_j x^j \qquad (5)$$

named discrete average approximation.

Approximation by Spline functions [6], [7], [8]. The obtained cubic Spline functions are named natural, or as the form:

$$S_{3,0}^{'}(x_0) = f'(x_0); \quad S_{3,N-1}^{'}(x_0) = f'(x_{N-1}); \qquad (6)$$

the functions resulted in these conditions being named strained cubic Spline functions.

Approximation by rational fractions (continuous fractions) [9], [10]. According to the specialized literature, a continuous fraction is defined as an expression as the form:

$$a_0 + \cfrac{b_1}{a_1 + \cfrac{b_2}{a_2 + \cfrac{b_3}{a_3 + \ldots}}} \qquad (7)$$

where $a_0, a_1, \ldots, a_n, b_1, \ldots, b_n$ are named the elements of the continuous fraction.

Approximation of the exponential curves [3]. In case of some exponential curves the approximation by polynomial functions is made with high errors, reason for that there was tried other approximation methods satisfactory from the point of view of error, being obtained for these curves through using of one approximation function as the form:

$$y(x) = \sum_{i=1}^{m} c_i \cdot e^{-\lambda_i \cdot x} \qquad (8)$$

with the condition $m \leq n$, therefore the number of sum terms smaller than the number of interpolation nodes.

III. OPTIMAL APPROXIMATION OF DATA OBTAINED AFTER SPORTSMEN TESTING

The "model" and "modeling" notions reached deeply in the sport theory and practice. The functions (cognitional, cultivational-educational, instrumental and normative), accomplished by the models used to solve the problems of the sport theory and practice, can have various characteristics. The efficient management of the training process is coherent with using of various models. V. Achim affirms that "the modeling will progressive became one of the most important principles of the sport training", while B.N. Şustin (1995) considers that the implementation of the modeling in sport represents an efficient and objective way to obtain sportive performances.

The models used on sports are divided in two basic categories: in the firs are included the models which characterize the structure of the contest activity namely those that hint at various aspects of the sportive training, the morpho-functional models, that reflect the morphological particularities of the human body, therefore assuring the reaching after the level requested by the sportive performance. In the second category are included models which reflect the continuity and the dynamics of the sportive performance establishing and of the short, medium, long and very long time plane planning and the models of various training exercises with the foresight of their complexity.

The general models reflect the characteristic of the object or of the process, obtained based on the study of a large sportsmen batch having an exactly sex, age, weight and which practice a certain kind of sport. To these models join, for example, the contest activity models at cross and swimming, the functional models of the basket-ball-men or hand-ball-men, the performance models on skiing and foot-ball, etc.

The group models are made based on a study of a sportsmen ensemble (or team), being different through a specific index in the category of each kind of sport, for example: The models of the "pass in five" technical-tactical actions at hockey, the models from the contest action of the wrestler and swimming etc.

The individual models are elaborated for each sportsman in part and these are based upon the data of the long researching of the separate training of the sportsman and upon his reactions to various tasks, etc.

The sportsmen performances are evaluated through periodical tests. Based upon these tests we can draw conclusions referring to the way in which the sportsman answered to a certain training program, to the parameters which can be increased, to the accumulated tiredness level. To extrapolate these data in the sight to aim at the next

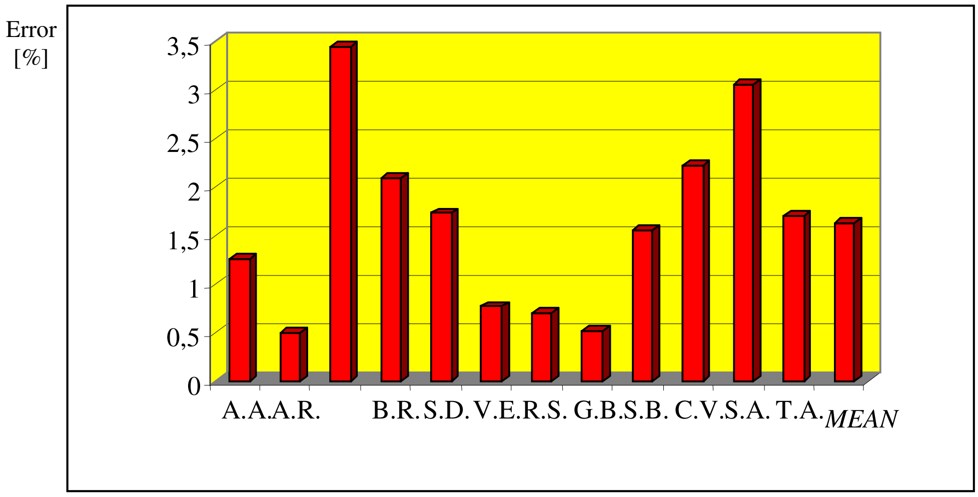

Fig. 1 The approximation errors in case of evaluation of the sportsmen's performances durring of the 8th month beginning with the training start maintaining on an established direction event

evolution of the sportsman and to predict some next performances it must to find an evolution law for values controlled periodically till a certain moment. This prediction can be made for one or more tests, at the middle of the training period, so that the evolution from the second part or from the end of this period could be instituted, in order to prepare the new training program.

From the specialized literature results that the sports forecasting problems centers round the using of the various forecasting methods (extrapolation method, modeling method and examination method). The forecasting made using the extrapolation method allows to form the sportive results on the hierarchical system on the basis of the study of some adequate laws from the previous period. The forecasting accuracy can be right if the forecasting period is shorter and if the data are more.

The extrapolation is very frequent used as a method to model and to obtained expertise data; therefore it will have in the future a more and more wide applicability on sports result forecasting. The forecasting applicability expectation will be closer by reality if it will be used prompter, more efficient and if it will use the informational technology possibilities, with the help of that will be processed and analyzed the sportive results obtained during a training macro-cycle or more macro-cycles even an Olympic cycle.

We propose, for determination of the optimal approximation method, to use relative error in the respect of previous definition, in computing considering the real value of the measured quantity as being the value of the evaluated quantity, it being measured into a certain number of points n with the help of some precision methods, and the measured value, in the respect of the previous definition, being the value computed in the respective input points using the approximation points determined through the previous presented methods. This methodology is in actual fact the same with that used to establish the precision class of the measurement devices, corresponding to the metrological norms.

In Fig. 1 the approximation error is graphical presented in case of evaluation of sportsmen's performances in the 8th month form the beginning of the training, in case of extrapolation of data from the first four months and of the comparison of these with the values determined at the end of period for the 50 m crossing.

From the presented data we observe only three cases in which the error exceeds the value of 2%. If we make a comparison with the measurement instruments, which normally assure an error of 2,5%, we can say that we obtained through extrapolation errors good enough. To mention that the values average is 1,5%.

To obtain a values average around the value of 1% is needed the re-evaluation of the functions which led to the two significant errors and the identifying of the graphical shape that needs an adjustment of the approximated values.

IV. CONCLUSION

On the base of the above presented, we can make important conclusions concerning of the value exploration and the prediction of some performances of tested sports:

- from the achieved graphics, where we represented the linear and exponential interpolated characteristics for each sports-girls, we observe the satisfactory way in which the exponential function approximates the obtained data (the exceptions are given by the very great performance leaps of some sports);
- the grouping of the initial and final values within the framework of tests divide them into three categories: test at which the obtained initial values are compact, the final values being dispersed (example: the tests for evaluation of the anaerobic speed), tests at which the obtained value, after a training period, are more grouped, then we obtain a homogeneity of the group (example: the establishing of the capacity to control de minimum speed, the test for heart frequency) and test that lead to a changing of the sports-girls values but with the keeping of the difference between them (example: the Wells and Dillon test, the standing long jump).

From the experiments and the tests made by authors, which numbered a very important volume of results and which include the previous representations, we can make the following conclusions concerning to the extrapolation error, using the exponential functions:

- the evaluation using exponential functions is made for training periods big enough, so that to include at least a discreet period of individual evolution, in other words to approximate the leap evolution for a certain test;
- the evaluation of the sportsmen which are trained for high performance (at the representative teams level) will be not made using extrapolation functions, because the person

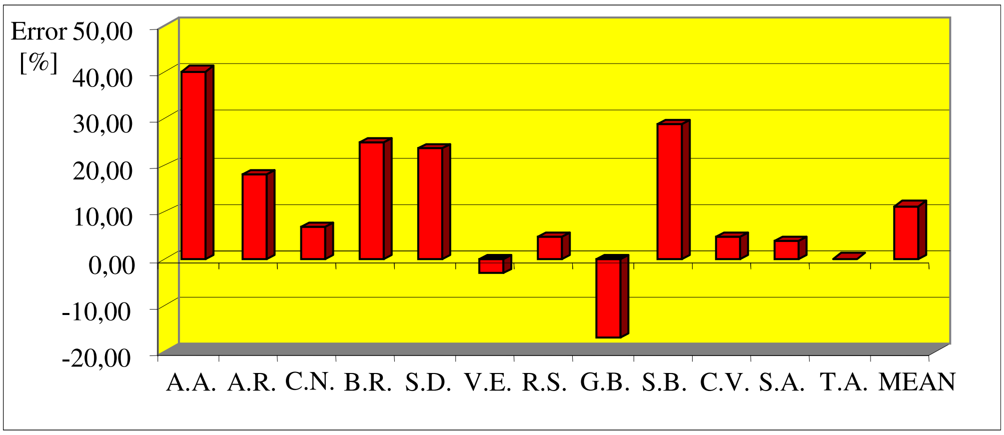

Fig. 2 The approximation errors in case of evaluation of the sport-girls's performances during of the 8^{th} training month for floating event

evolution cannot keep a certain shape of the function from one person to another or from one test to another, in these cases the evolution having on the base functions with discontinuities and leaps;

- the human nature offers a no-uniform growing not only from person to person (reason for that it was determined one approximation function specific to each sports-girls) but for the same person also, at the different time intervals and for different training programs. From this reason there was obtained, in dispersed cases, great approximation errors, either negative (in case in which the dynamic was more then great in the first part of the training period) or positive (for the case of the sports-girls which, in the first part, have not performances great alike that in the second part of the training);
- we can observe a approximation by missing for some events (speed crossing, floating) – that means a sub-estimation by extrapolation, or approximation by addition at another events as long jump and high jump, performed by predictive overestimation. From here results the evolution characteristic of some sportive performances during the training period;
- the human performances modeling, using the continuous mathematical functions, is the source of the appearing of some performance evaluations difficult to anticipate for some sportsmen. Thus, for sports-girl C.N. we obtain a maximum positive error at the speed crossing and a maximum negative error in group for long jump, fact due to the different evolution in the first respectively second part of the training period. This fact has on the base either the specific characteristics of the human body or the accent putted, during the training period, on the different aspects of the training depending on the obtained intermediary results;
- over than 50% from the obtained errors integrates oneself with the accepted scientific domain, that represents a satisfactory prediction, if we take into account the complexity of the alive nature and the distribution with fractal characteristic of the performances evolution of man;
- the used function at extrapolation assures satisfactory errors also in case of the approximation using two, maximum three points which divide the interval in equal subintervals;
- for the sportsmen at which we obtained great errors in case of extrapolation, it is recommended the introduction into the approximation functions of one correction coefficient, if their evolution can be observed.

We can mention that, in the actual training process, the modeling method became a basis method, a working instrument of trainers.

Based on the previous presented we can make some important conclusions concerning the extrapolation of the values and the prediction of some performances at the tested sportsmen level:

- taking into account the human nature, the its evolution and the specific of the sportsmen training, for medium and short times, the evolution by leaps can be modeled through a continuous curve that offers the extrapolated values only to the end of the interval;
- after the effected tests, we arrived at the conclusion that for the considered time interval the best approximation by extrapolation is achieved with exponential functions;
- there are tests (of height, of weight) at which the testing period not leads each time at the obtaining of the significant evolutions, case in that the approximation function is a constant. This fact can be explained by that the period after we notice the quality leap at most of the sportsmen is greater then the time between the achieved tests.

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